

Reprise of a “Dense and Tense Story”

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Consider the following problem.¹ Two uniform cubes have sides of length L . Cube 1 has volume mass density ρ_1 , while cube 2 has density $\rho_2 > \rho_1$. Their average density, $\rho = (\rho_1 + \rho_2)/2$, is equal to that of an incompressible fluid filling a beaker. The two cubes are glued together and fully immersed in the fluid with the lighter cube 1 positioned directly above cube 2, such that the interface between them is at depth H . Suppose that the glue has a density equal to that of the fluid, so that the combination of blocks and glue is overall neutrally buoyant in the fluid. Denote by F the maximum tensile force that the glue can withstand before tearing apart. Under what conditions will the cubes break apart (resulting in cube 1 rising to the surface and cube 2 sinking to the bottom)?

An incorrect analysis² assumes that the interfacial force acting on block 2 can be taken to merely be the upward glue force F , so that

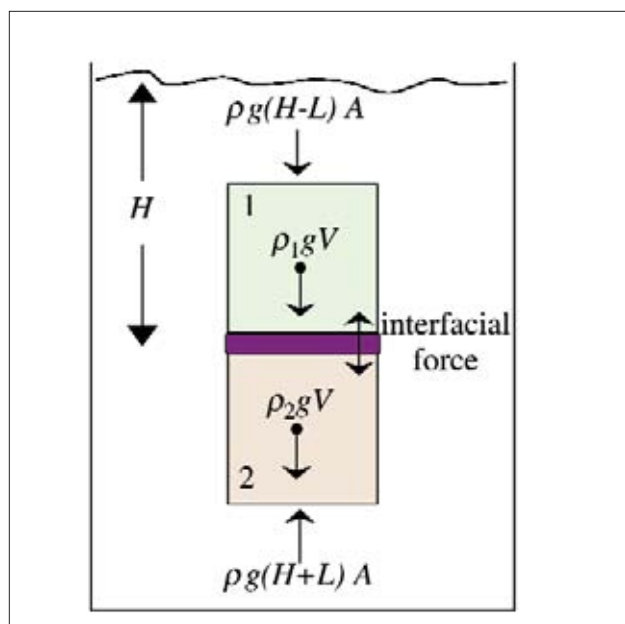
$$\rho g(H + L)A - \rho_2 gV + F = 0, \quad (1)$$

where the volume of a cube is $V = L^3$ and the area of a face is $A = L^2$. Keeping Newton's third law in mind, similar reasoning for cube 1 gives

$$\rho g(H - L)A + \rho_1 gV + F = 0. \quad (2)$$

Adding Eqs. (1) and (2) and substituting $\rho = (\rho_1 + \rho_2)/2$ predicts that the blocks will break apart at a depth

$$H = \frac{(\rho_2 - \rho_1)gV - 2F}{(\rho_1 + \rho_2)gA}, \quad (3)$$



Free-body diagram for two cubes of differing densities glued together and neutrally buoyed up against gravity by submersion in an incompressible fluid.

which corrects an obvious minus sign error in Ref. 2.

The problem here is that each of the three terms in Eq. (2) is positive. This would imply that cube 1 *cannot* be in equilibrium: There is no upward force to balance the three downward forces. Thus, this analysis is flawed.

A number of my colleagues have suggested fixing the analysis as follows. Instead of an upward glue force F , suppose we only have a downward normal force N exerted on cube 2. In that case we would rewrite Eq. (1) as

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$$\rho g(H + L)A - \rho_2 gV - N = 0. \quad (4)$$

The analogous equation for cube 1 is

$$\rho g(H - L)A + \rho_1 gV - N = 0. \quad (5)$$

These equations do not apparently depend on F . The suggested interpretation is that the cubes will not tear apart from each other *regardless* of the difference in densities of the two blocks!

This is intuitively unreasonable. It is true that if no fluid seeps into the interface, the blocks cannot separate. This really begs the question: Will the glue fail, thereby permitting fluid to penetrate between the blocks? Clearly, it is necessary to *explicitly* account for the glue force in the analysis in order to answer this puzzle.

So I suggest that we combine both of the ideas above and model the interfacial force on block 2 as the *sum* of a downward normal force and an upward (tensile) glue force,

$$\rho g(H + L)A - \rho_2 gV + F - N = 0. \quad (6)$$

Similarly for block 1, the force balance becomes

$$\rho g(H - L)A + \rho_1 gV + F - N = 0. \quad (7)$$

Now suppose that the glue is on the verge of failing, so that fluid has fully seeped between the blocks except at a few remaining spots (of negligible cross-sectional area) where the glue is still attached. In that case, the normal force pushing block 1 upward and block 2 downward is principally exerted by the fluid,

$$N = \rho gHA. \quad (8)$$

Substituting Eq. (8) into (6) and (7), and then adding this latter pair of equations together, now implies that the two blocks will *eventually* come apart if

$$F < \frac{W_2 - W_1}{2}, \quad (9)$$

where the weights of the cubes are $W_1 = \rho_1 gV$ and $W_2 = \rho_2 gV$. In contrast to Eq. (3), this predicts that

there is no dependence on the depth of immersion. On the other hand, in contrast to the analysis using Eqs. (4) and (5), we now have a clear prediction that a strong glue will hold the blocks together and a weak glue will not. I decided to test these two predictions experimentally.

Since it is tricky to set up the actual situation in the figure, I instead replaced the lower block with a cup to which the upper block was glued. A rectangular prism of low-density plastic (i.e., having a specific gravity less than unity) was glued to the bottom of a deep, disposable cup using rubber cement. (This adhesive can be reproducibly re-glued many times in succession.) In order to ensure repeatability from trial to trial, the object was always repositioned at the center of the cup bottom, and a specific metal weight was placed on top of it briefly to bond the glue to the same extent in each case. No fatigue of the glue was observed, as checked by running the experimental trials repeatedly in random order.

Water was gently poured down the inner edge of the cup for each trial to prevent surges from jarring loose the plastic block. If the plastic cylinder was half submerged, it did not break loose when watched for several minutes. However, when water was filled to just above the top of the object, it broke free and floated to the surface in about 10 seconds. Finally, if water completely filled the cup, the object again broke free, but not any faster than when the water just barely covered it. Also, if a substantially heavier weight was used to initially press the object down into the glue and stick it more firmly, the plastic prism was not observed to tear free of the bottom for any subsequent depth of submersion.

The above results are consistent with my solution above, but do not agree with the predictions of the preceding two models. Incidentally, it is interesting to watch the manner in which the glue fails. The plastic tower is observed to slowly tilt in one direction until it is only attached along an edge. Finally it breaks free of this edge and fully flips onto its side as it rises to the surface. A reasonable interpretation is that water slowly seeps into a weak spot somewhere around the periphery of the glue interface and pries the cylinder up in a manner reminiscent of pulling plywood up with a claw hammer.

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References

1. Adapted from “Physics Challenges for Teachers and Students: Dense and tense story,” *Phys. Teach.* **40**, 48 (Jan. 2002).
2. “Physics Challenges for Teachers and Students: Dense

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